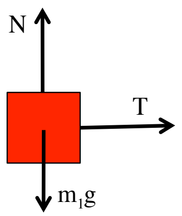
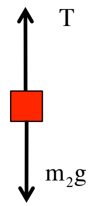
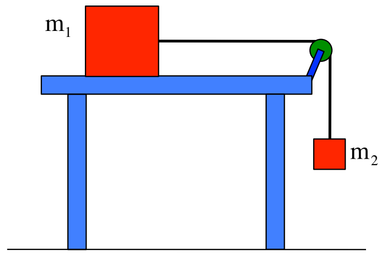
Problem 5.28

a.) Free body diagrams:







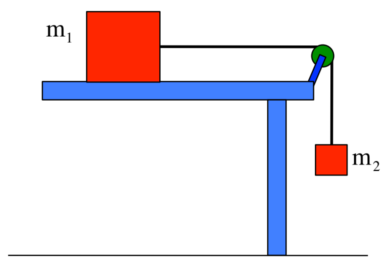
Notes:

1.) Remember, ideal pulleys just change the direction of tension force.

2.) As the algebraic variables are just denoting magnitudes, using "T" in the horizontal in one f.b.d. and "T" in the vertical in the other is not contradictory.

3.) Remember, sliding a vector on a f.b.d. is an OK operation as long as you don't change the orientation and there aren't torques involved.

b.) Determine the acceleration (we'll use the Quick and Dirty approach):



The only force motivating the system to accelerate is m_2g , and the total mass is $m_1 + m_2$.

Soooo:

$$\underline{\sum F_{acc}:}$$

$$\Rightarrow \quad m_2g = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{m_2g}{(m_1 + m_2)}$$

$$\Rightarrow \quad a = \frac{(9.00 \text{ kg})(9.80 \text{ m/s}^2)}{((9.00 \text{ kg}) + (5.00 \text{ kg}))}$$

$$= 6.30 \text{ m/s}^2$$

It is interesting to note that the Solution Manual for the textbook used the Formal approach to solving this problem. Just to be complete, and for those of you who tried that approach, I'll do the problem that way and I'll do it the way you might do it on a test should you decide to use that approach on a test (that is, I'll follow but won't enumerating the steps).

Before we get started, though, a few more notes:

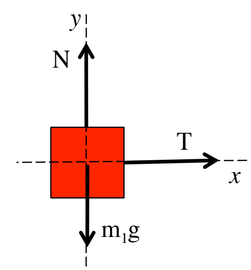
1.) We have two individual masses each of which has its own acceleration. The accelerations as vectors are different in the sense that their directions are different, but they are the same in the sense that their magnitudes are equal.

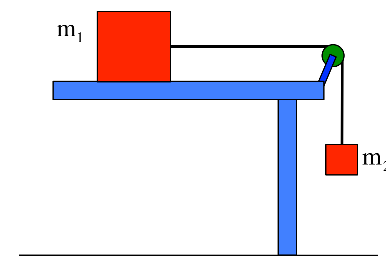
2.) Assuming the tabletop mass accelerates to the right, which it does, the hanging mass accelerates DOWNWARD. If, for the hanging mass, we take the positive direction to be UP in the direction of the tension force, the h.m. acceleration will be DEFINED BY OUR AXIS to be in the negative direction.

3.) If, when we write out "ma" for the hanging mass, we put a negative sign in front, that unembedding of the acceleration vector's sign will make the "a" term a magnitude in that expression, just like the "a" term found in the tabletop mass's N.S.L. expression. With that, solving the two N.S.L. expressions simultaneously is eeeeasy (or, at least, doable).

So here goes with the *Formal approach*:

f.b.d. and axes on m_1:





Soooo (and noting that numbers aren't usually put into these problems until the end):

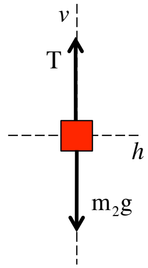
$$\underline{\sum F_x:}$$

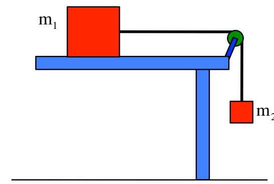
$$T = m_1a$$

where "a" is the magnitude of the block's acceleration. (Notice that both "T" and "a" are in the positive direction, relative to our coordinate axis, so both are written as positive terms in the equation.)

(Notice that I've denoted the vertical axis as "v" and the horizontal axis as "h"--I've done this to circumvent confusion with the other f.b.d.'s axis system.)

f.b.d. and axes on m_2:



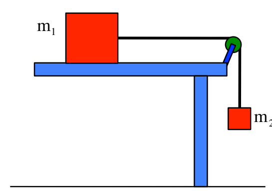


Soooo,

$$\underline{\sum F_v:}$$
$$T - m_2 g = -m_2 a$$

where "a" is the magnitude of the hanging mass's acceleration. (Note that "T" is in the positive direction while "mg" and "a" are in the negative direction, thus their assigned, umembedded signs in the N.S.L. equation.)

Combining the two equations and eliminating the tension "T" using our first N.S.L. equation, we get:

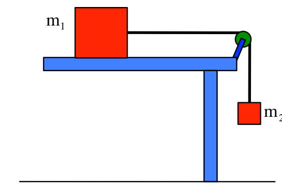


$$T - m_2 g = -m_2 a$$
$$\Rightarrow \quad (m_1 a) - m_2 g = -m_2 a$$
$$\Rightarrow \quad a = \frac{m_2 g}{(m_1 + m_2)}$$
$$\Rightarrow \quad a = \frac{(9.00 \text{ kg})(9.80 \text{ m/s}^2)}{((9.00 \text{ kg}) + (5.00 \text{ kg}))}$$
$$= 6.30 \text{ m/s}^2$$

Yeeha! We get the same acceleration as with the Quick and Dirty approach, but with considerably more pain . . . hence an excellent example of why the Quick and Dirty approach is the way to go if you can.

Minor Point: What would have happened if we had not manually placed that negative sign in front of the "ma" term in the hanging mass's N.S.L. expression?

In that case, we'd have written:



$$\underline{\sum F_v:}$$
$$T - m_2 g = m_2 a$$
$$\Rightarrow \quad (m_1 a) - m_2 g = m_2 a$$
$$\Rightarrow \quad a = \frac{m_2 g}{(m_1 - m_2)}$$

If you had done this without numbers, something should have screamed at you at this point. Specifically, what would happen if the two masses had been the same size (which could have happened)? In that case, the numerator would have gone to zero and the expression would have blown up. There is clearly something wrong here!

If you ever run into something like this, it means you've dropped the negative sign in the "ma" part of one of the N.S.L. equation.